**Formal Properties of Green’s Functions**

Still more…

**Operator single particle Green’s function**

Operator GF’s are a useful concept. They make it convenient to put single particle Green’s functions in different bases. Consider a generic single particle Green’s function, like the causal one:



where the bracket can stand for a commutator, or nothing or whatever. Then the operator Green’s function would be defined as:



where |j> is the single particle eigenstate associated with operator bj – related in the manner of |r> and ψ(r). If a GF in a particular basis is desired, say the momentum basis, then we just form,



**Differential equations obeyed by single particle GF’s**

Let A = ψ, and B = ψ†, and then we’ll investigate the differential equations obeyed by the Green’s functions. Suppose that we have a spin-dependent single particle potential:



which in 2nd quantized notation is:



In the ‘2nd quantization in Various bases’ file, we saw that the time development of ψσ(x) is:



So from the definition of the respective green’s functions, and the equation of motion of ψ, we can work out the equation of motion of G.



So,



If the single particle potential were not spin-dependent, i.e., if it were proportional to δσσ´´, then of course this would decouple the spin parts. Boundary conditions for the PDE would be that GR(x,t;x´,t´<t) = 0. We should find the same equation for GA, though with opposite boundary conditions. For the causal GF, we have:



Recognizing the commutator gives us the δσσ´δ(x-x´) function, then we get the same equation as for GR,A,



(though we could’ve anticipated this result from the algebraic relationships) Note the similarity to the Schrödinger equation. What about its boundary condition? We should have GC(x,t;x´,t´=t) = <ψ(x)ψ†(x´)> ?. The >, < green’s functions don’t have the δ’s.



Let’s step up to an interaction potential (spin – independent):



In second quantized notation this would be:



and consider the differential equation obeyed by GC(r,t;r´,t´). We’ll use the equation of motion of ψ again, which in this case is (from the 2nd quantization in various bases file):



where



Using this we can work out the equation of motion of the Green’s function:



and we can write this in a manner similar to the distinct particles evolution equation:



where the interaction term is defined as:



and n is the density operator of course. So we see that this effect of the interaction is to

add an effective term to the Hamiltonian. This is a possible starting point for a non-perturbative calculation of the Green’s function. For instance, we could do a mean field approximation and take out the density operator. Let’s presume spin-independent V1 too. Then we have:



which would give us:



By using the MFA on the ψ PDE itself, we can see that we’d also have:



The >, < green’s functions don’t have the δ’s. The differential equation obeyed by the operator GF’s will, in the mean field approximation (which is exact if no two-particle interaction) look like:



where H(r,t) = H1(r,t) + <H2(r,t)>. And so we have, generally:

